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MULTIPLE COHERENCE OF LONG PERIOD NOISE AT LASA

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MULTIPLE COHERENCE OF LONG PERIOD NOISE AT LASA

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ABSTRACT

Multiple coherence gives a quantitative measure versus frequency of how well a linear combination of n input channels can match the (n + 1)st channel in a seismic array. If the inputs can match the output exactly, then the multiple coherence is unity and only n channels are necessary to describe the noise field. This report shows multiple coherence versus frequency with 2 to 9 input channels for long period, vertical component noise fields at LASA.

Over the 7 to 20 seconds period range the multiple coherences on the samples tested were greater than .65 showing that 65% or more of the noise at a center channel is predictable by other seismometer outputs in the array. This level of multiple coherence requires 8 to 9 input channels. Multiple coherence with fewer inputs and ordinary coherence between pairs of channels are much lower.

From the samples tested, which all produce multiple coherences quite similar to each other, we conclude that at least 9 input channels are necessary to adequately describe the long period noise at LASA.

1. INTRODUCTION

Most basic data processing techniques for signal enhancement or identification depend upon the structure of the noise within the seismic array. If some of the coherent noise is due to site characteristics such as consistently coherent noises from particular directions, then techniques using multiple coherence will help to isolate these consistent linear relations. Many optimum filters for estimating the signal take account of these linear relations implicitly by weighting with the inverse of the spectral noise matrix. However, one cannot tell whether the coherent noise involved is due to noise generating events which cannot be predicted or controlled. Thus, the filters must be recalculated over a period of noise recording immediately prior to the arrival of each single signal. Part of the coherent noise generated within the array may be due to various causal factors for a particular array. If so, we can learn something about these factors by examining the linear relations between the various array elements. A potential benefit here is that a consistent linear model relating the different sub-elements would eliminate the need for computing a different set of filter coefficients for each event.

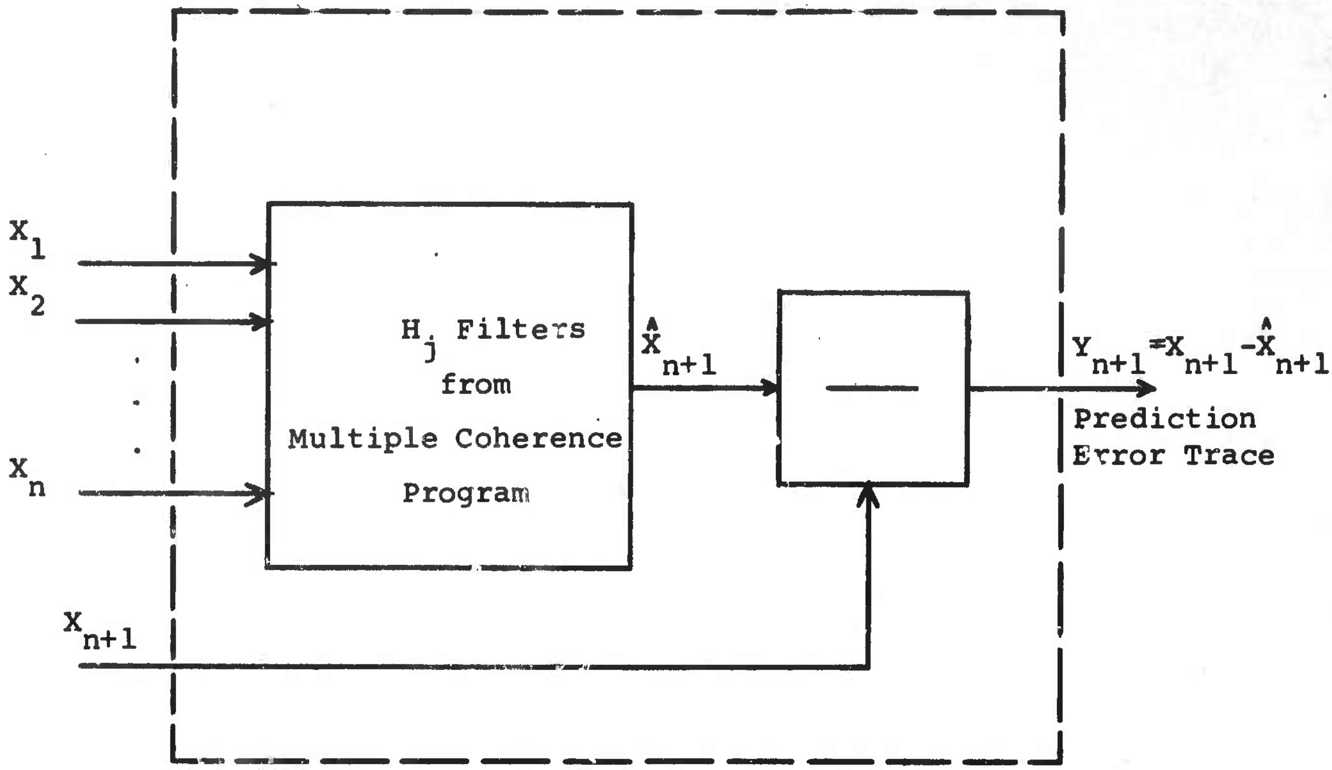
The multiple coherence function can indicate how many seismometer outputs in an array are necessary to properly determine the seismic noise field. If there are n independent seismic noise components, then the multiple coherence function would be unity when (n + 1)st seismometers are placed in an array to measure seismic noise records. If part of the background is

composed of incoherent noise, then the multiple coherence function would indicate the percentage of coherent noise present and the number of seismometers necessary to define this coherent noise. The filter relations determined by the multiple coherence computations can then be used in array summation to bring the noise into destructive interference.*

This analysis does not guarantee that such optimum processing is possible. For example, if the noise and signal propagation characteristics across the array are identical, no velocity filtering scheme can be expected to separate the two even though the multiple coherence might be unity.

The multiple coherence function is the frequency domain equivalent of the prediction error filter in time. If n input seismic traces predict the (n + 1)st trace in an array completely, then the multiple coherence will be unity and a prediction error filter could be used to exactly predict this (n + 1)st output. In fact, linear filter relations derived by the multiple coherence program produce an estimate of the (n + 1)st trace which, when subtracted from the actual (n + 1)st trace, given a prediction error trace. Thus the combination of the filter derived in the multiple coherence program and the subtraction operation produces a prediction error filter as shown in the following diagram.

*For the mathematical description of the multiple coherence computation, see Appendix I.



Prediction Error Filter

The first objective of this study is to use the multiple coherence function to estimate the degree of predictability of the long period noise field at LASA. The result in turn should tell us how much the noise power should be reduced by optimum filtering (e.g., maximum likelihood or Wiener filters) if the filters were theoretically ideal.

The second objective is to determine from multiple coherences the number of independent components comprising a given noise field and the percentage of incoherent noise which cannot be cancelled by any kind of multichannel filtering.

The third objective is to determine the stationarity properties of the noise field. We accomplish this by applying the multiple coherency program to three different time samples from the same array. Then the multiple coherency filters derived

from the first time sample are applied to the other two time samples. If the filters derived in the first time sample have done a good job of predicting the noise field in all three samples, then the data are said to be stationary. On the other hand, if the filters from the first time sample do a progressively poorer job of predicting the noise in the other samples relative to the filters associated with those samples, then the noise is non-stationary to some degree. This deterioration in predictability of the multiple coherence can quantitively measure the non-stationarity of the data**.

**For a theoretical discussion of the stationarity computation, see Appendix II.

2. DESCRIPTION OF DATA

We computed the multiple coherence of long period LASA data for the vertical components only. These seismometers are located at the center of each subarray. Consequently, all computations are intersubarray coherences with a maximum of 21 channels available.

The sampling rate for the data is one sample per second. The number of points in the sample varied between 1200 and 4800, from 20 minutes to 80 minutes of data. One frequency range computed varied from zero to .25 cps with a frequency interval of .0125 cps. A second frequency range varied from zero to .20 cps with a frequency interval of .008 cps.

The multiple coherence program has a capacity of 9 input and one output channels. We varied the number of input channels from 2 to 9 wherever possible. In some cases, spikes, dead traces, and instrument malfunctions prevented our obtaining more than 6 or 7 useable input channels. The ordering of the input channels was from the outermost subarrays toward the inner subarrays in every case.

We examined three noise samples. The first was from 9 September 1966 and contained 4759 points. The second from 11 December 1966 contained 1200 points and the third from 5 January 1967 contained 3957 points. We used the 9 September 1966 sample to test variations in the sample lengths and number of lags. We used this sample for the stationarity test also by breaking it into three equal lengths.

3. RESULTS

Multiple Coherences

Figures 1 and 2 show the multiple coherences versus frequency for the three time samples tested. Figure 1 shows the multiple coherence for the 9 September 1966 sample with 2 to 7 inputs and for the 11 December 1966 sample with 2 to 6 inputs. The figure shows a diagram of the array elements chosen. The output in both cases is channel A0. The September sample is for 3957 points and 120 lags and the December sample is for 1200 points and 80 lags. Figure 2 shows the multiple coherences for the 5 January 1967 sample computed with two different lags; the first at 200 lags and the second at 120 lags.

All of the multiple coherences versus frequency are similar. The multiple coherence for all samples increases significantly with the increase in number of input channels. The sample for 5 January 1967 shows multiple coherences, with 8 or 9 input channels, of .65 and above for frequencies from .05 to .14 cps (from 7 to 20 seconds period).

The multiple coherences for 7 or fewer channels for the 5 January sample agree fairly well with the multiple coherences of December and September samples. This similarity indicates that the September and December samples would also show high multiple coherences over the same frequency range if 8 or 9 reliable input channels were available.

The multiple coherences for the January sample for 120 and 200 lags agree with each other. Even 120 lags at one sample per second is sufficient to uncover correlations between the

outermost subarrays and the output A0 provided the noise propagations are at velocities of 1 km per second or higher. Since we expect all noise propagation to have velocities greater than this limit, we would expect that the 120 and 200 lag cases should give essentially the same results.

Power Spectra

The spectra for the multiple coherence examples on Figure 1 and 2 are presented on Figures 3 to 7. Figure 3 shows the power spectra for the 9 September 1966 sample for all 4759 seconds. Figures 4 and 5 show the power spectra from the first and second third of this same sample. All three spectra are essentially the same. The vertical scale for all spectra plots are in relative power. At the time these recordings were made, long period calibrations at LASA were not available.

Figure 6 and 7 show similar power spectra plots for the 11 December 1966 and 5 January 1967 samples respectively.

Stationarity Tests

The multiple coherence program derives a set of n filters for the n input seismograms which together provide the best linear estimate for the (n + 1)st seismic trace. The difference between the observed (n + 1)st trace and the best estimate is the error trace. If the multiple coherence is unity, the prediction is perfect and the error trace will be zero. If we form the ratio of the error spectra over the observed spectra, we can get a measure of the reduction in noise power possible from the theoretical

optimum filters. Thus the db improvement as a function of frequency can be expressed as

$$db = 10 \log (error/observed).$$

The prediction error filters will do the best job in eliminating the noise background when they are applied to the noise sample from which they are derived. However, if the noise is stationary, the same filter could be expected to do nearly as well when applied to later time samples from the same array. Figure 8 shows the expected noise reduction in db when the prediction error filters that were derived from the first time sample are applied to the first 1586 second sample. In addition these same filters are applied to the second and third 1586 second samples.

The expected noise reduction from prediction error filters in the fitting interval is as much as 9 db at .064 cps (16 second period). Over the 7 to 20 second period range of high multiple coherence the average noise reduction in the fitting interval is about 4 db. This result is obtained with only 7 input channels. Due to the significant increase in multiple coherence with 8 or 9 inputs as shown for the 5 January sample, we would expect that the db improvement in the fitting interval would be somewhat better with more channels. Over the same 7 to 20 second period range, an average of approximately 1 db improvement is obtained outside the fitting interval. The noise reduction outside the fitting interval at .064 cps (16 seconds) is from 3 to 4 db.

Figure 9 shows the same computations as Figure 8 but with 150 lags computed in the correlation functions instead of 50 lags. The results of Figure 9 essentially agree with those of Figure 8 but show much higher variability. For multiple channel cases, 10% lags led to too much statistical variability to make the results reliable.

Statistical Variability

The examples in this section show the increase in the statistical variability of the computations with the decrease in the degrees of freedom. The (real) degrees of freedom are defined in terms of the number of sample points, the number of lags, and the number of input channels as follows:

$$\text{degrees of freedom} = 2 \times \left(\frac{\text{no. of points}}{\text{no. of lags}} - \text{no. of input channels}\right)$$

Thus we see that the degrees of freedom will decrease as we decrease the number of points in the sample or increase the number of lags or number of input channels. We note that the effect of decreasing the degrees of freedom is to increase the estimate of the multiple coherences.

Figure 10 shows an increase in multiple coherence with only two inputs when the degrees of freedom are decreased from 44 to 2 in several steps. In this case the 1200 points in the sample was held fixed and the degrees of freedom changed by increasing the number of lags.

Figure 11, a similar plot, is shown for the same data sample with 7 input channels. In this case the number of degrees of freedom is decreased from 34 to 2 in several steps. The actual computations for Figures 10 and 11 are shown in Table 1 where the multiple coherence is tabulated against frequency and number of lags. When the number of degrees of freedom become too small, the instability in the computations can cause the computations for the multiple coherence to lie outside the range from 0-100%.

The effect of too few degrees of freedom can be seen in another way. Figure 12 shows the multiple coherence versus frequency computed with 100 lags when the number of points in the sample was varied from 4759 down to 759, in several steps. Here again the multiple coherence increases as the degrees of freedom decrease until instabilities yield coherences outside the range from 0-100%.

When the degrees of freedom are 20 or higher, the computations are generally quite close to each other. For the results shown in Figures 1 and 2 we maintained the degrees of freedom in excess of 20.

4. CONCLUSIONS

1. Multiple coherences for all long period samples increase significantly with an increase in the number of input channels.

2. From mid-range of the long period pass band toward the microseismic frequencies (7-20 seconds period) the multiple coherences are greater than .65 with 8 or 9 input channels.

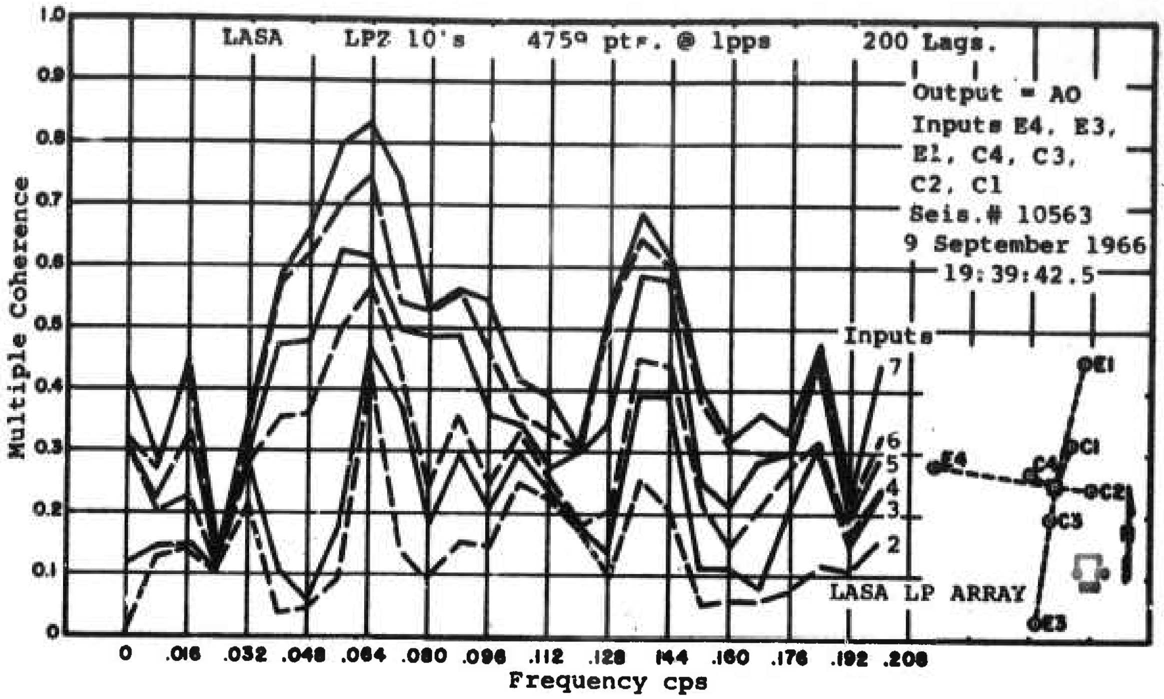
3. The expected noise reduction from a prediction error filter in the fitting interval is as much as 9 db at 16 seconds period. The noise reduction outside the fitting interval at 16 seconds period is from 3-4 db.

4. Over the 7-20 second period range, the expected noise reduction from a prediction error filter in the fitting interval is about 4 db. Over the same 7-20 second period range an average of 1 db is obtained outside the fitting interval.

5. The db improvement figures given above were computed when only 7 input channels were available. We estimate that the db improvement both within and outside the fitting interval would be increased with more input channels.

6. Over the 7-20 second period range the multiple coherences for all samples tested are quite similar to each other up to 6 or 7 input channels. Since the 9 channels available for one sample showed a significant increase in the multiple coherence over the 6 and 7 channel examples, we conclude that at least 9 input channels are necessary to adequately model the long period noise at LASA.

FIGURES



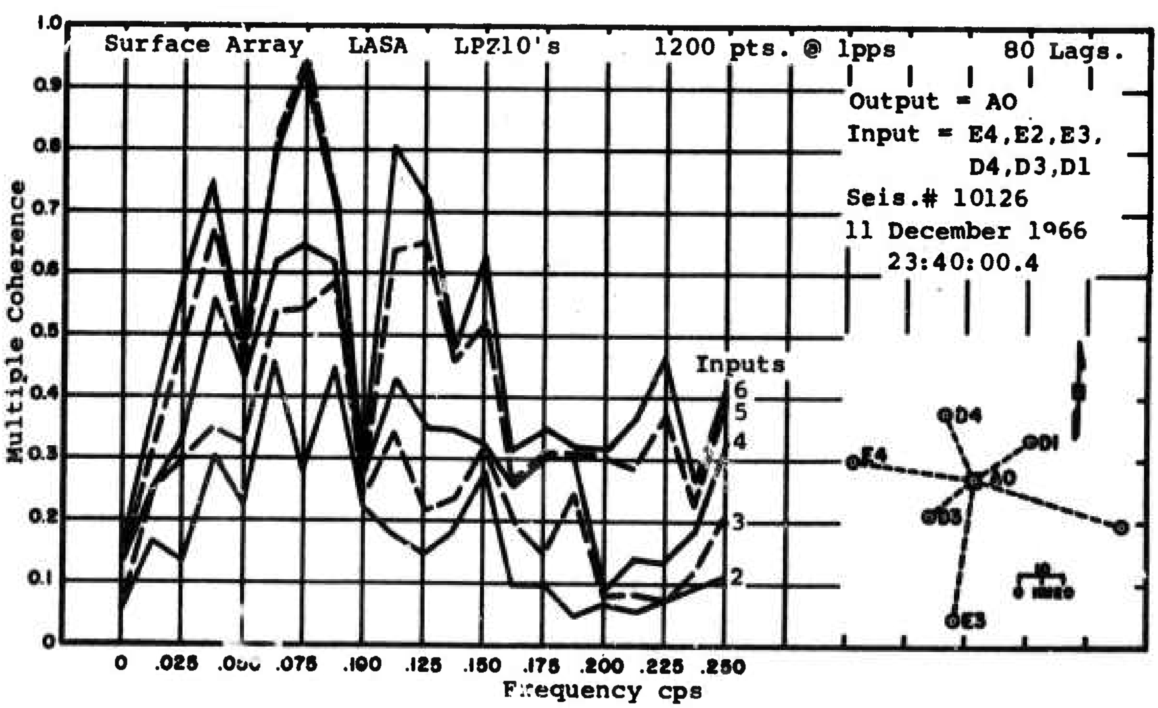
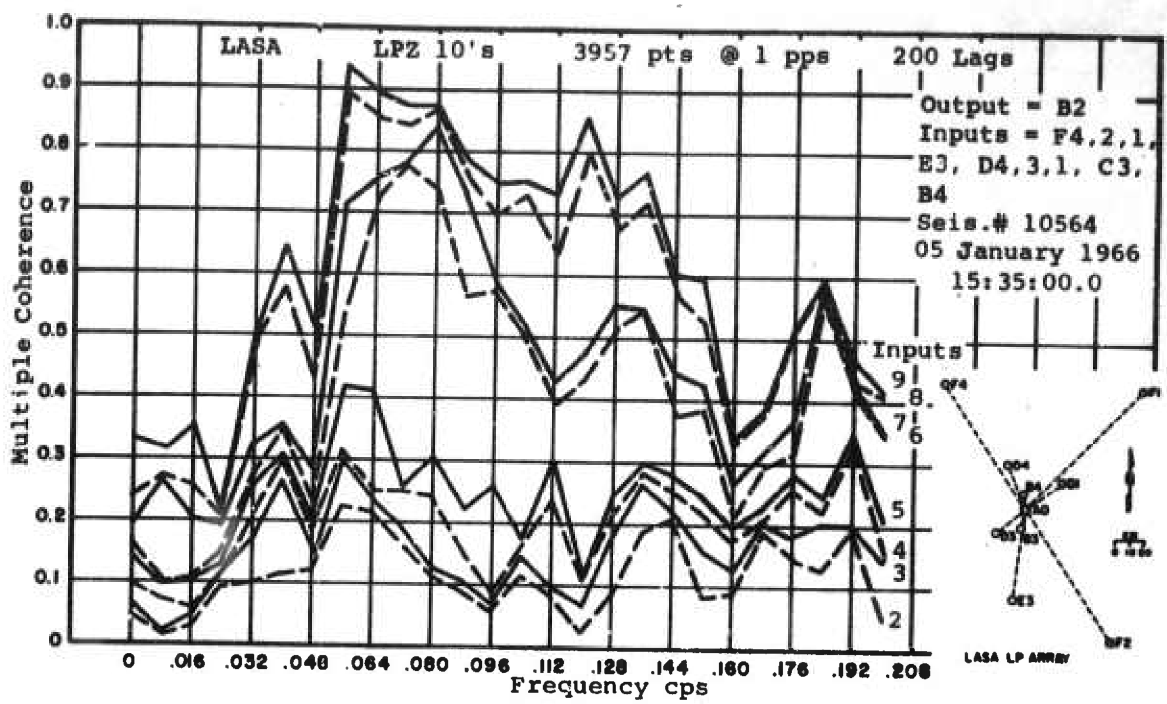


Figure 1. Multiple Coherence vs. Frequency for two LP-Z Samples from LASA.



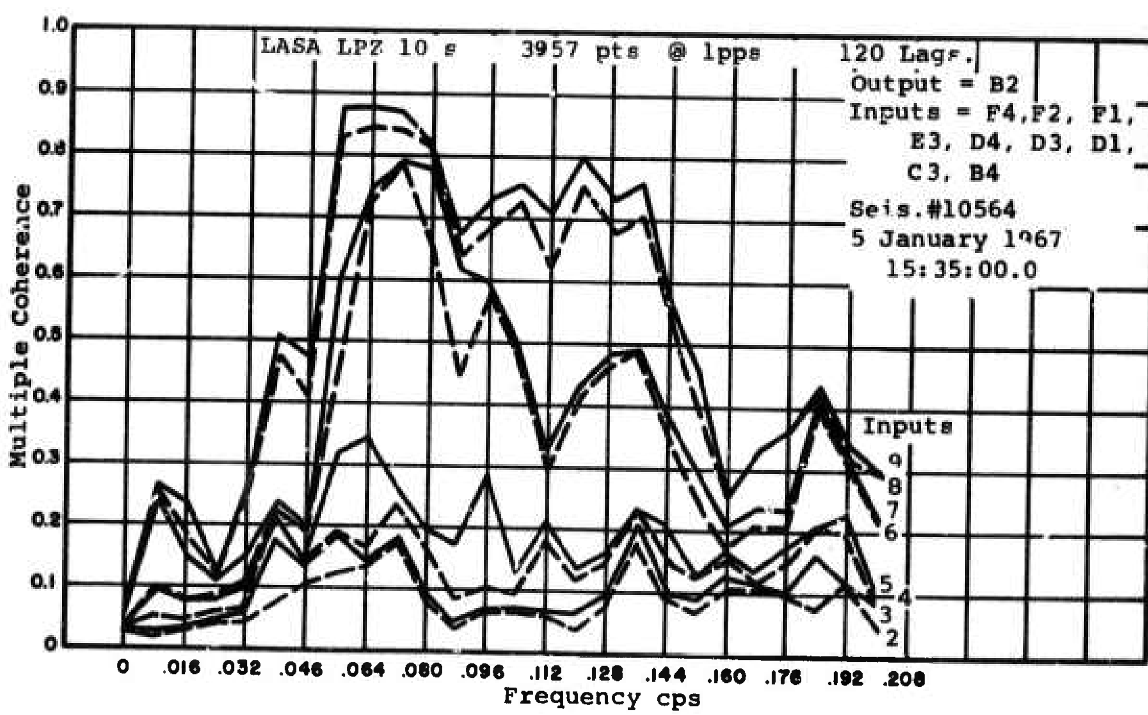


Figure 2. Multiple Coherence vs. Frequency for One LP-Z Sample from LASA Computed with Two Different Numbers of Lags.

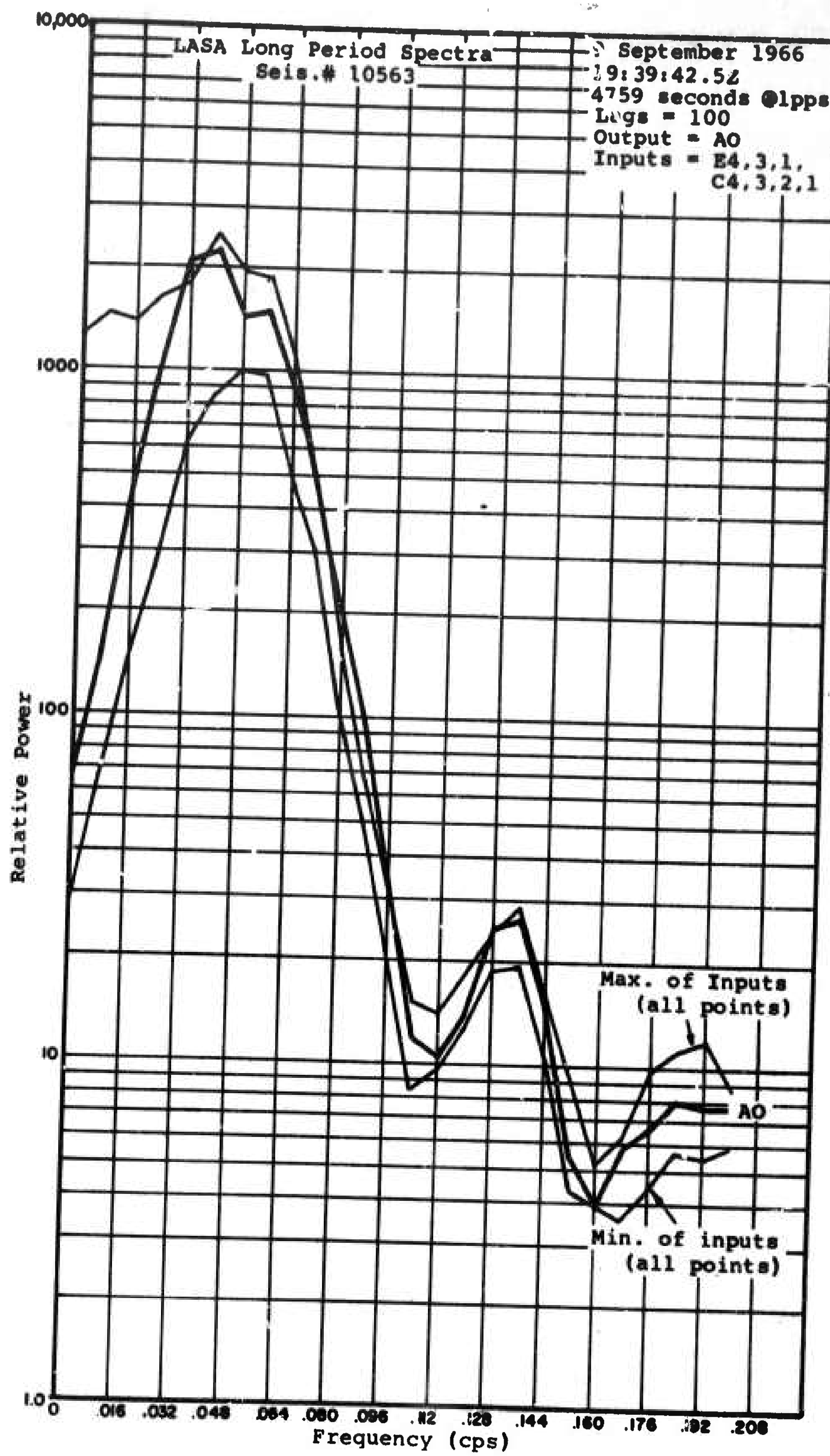


Figure 3. LASA Long Period Spectra for a 9 Sept 1966 Sample. The Figure shows the output spectrum and the range of all input spectra.

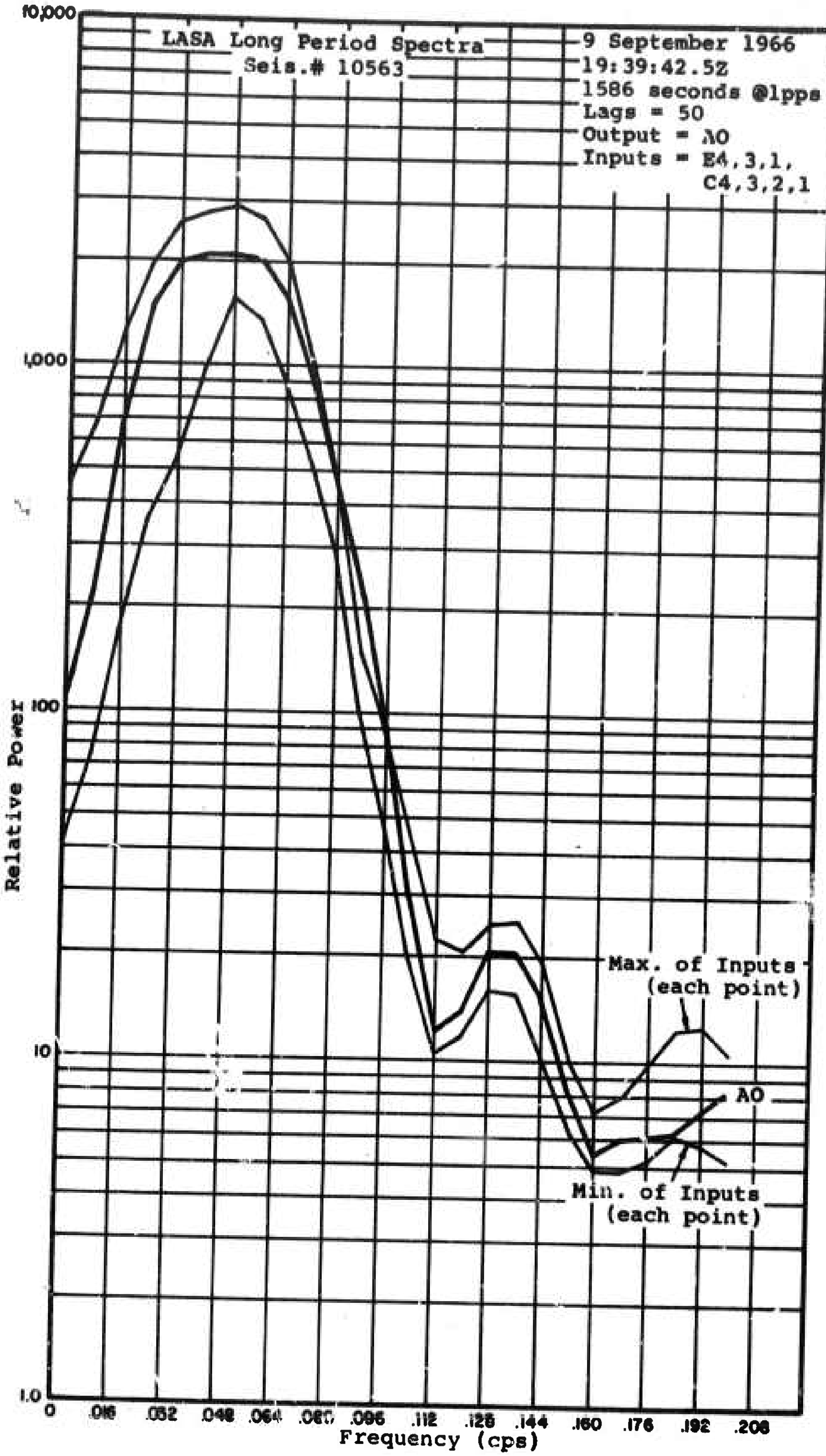


Figure 4. LASA long period spectra for the first third of the 9 Sept. 1966 sample shown on Figure 3.

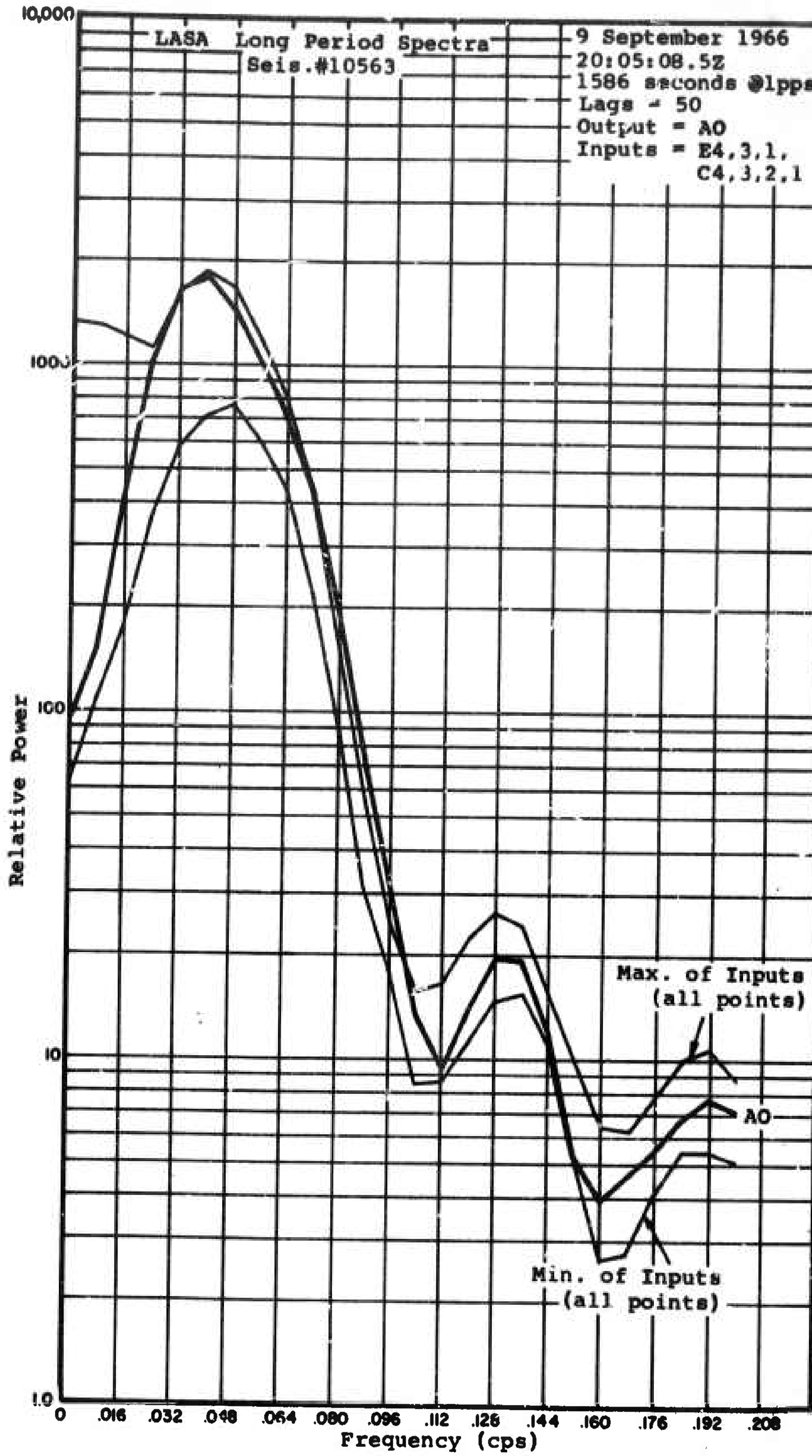


Figure 5. LASA long period spectra for the second third of the 9 Sept. 1966 sample shown on Figure 3.

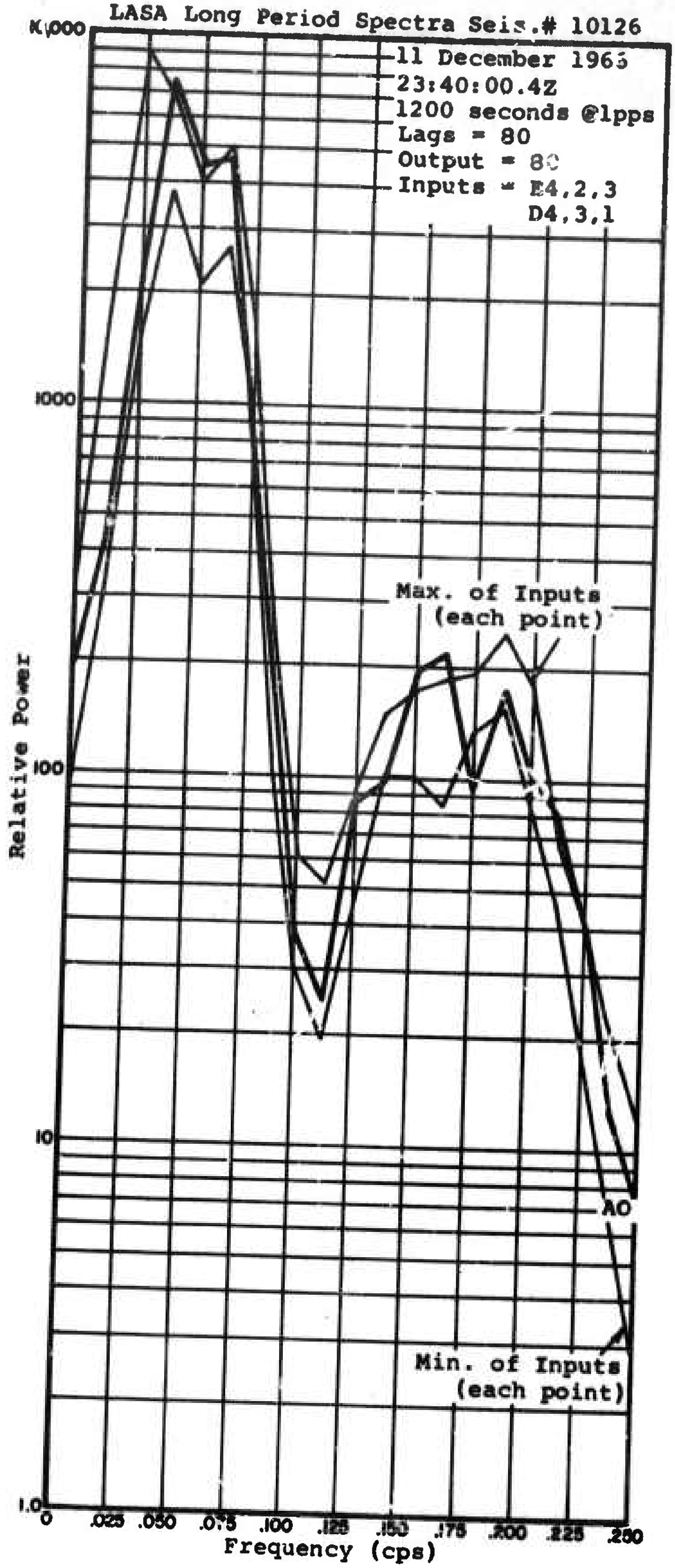


Figure 6. LASA long period spectra for a 11 Dec. 1966 sample. The figure shows the output spectrum and the range of all input spectra.

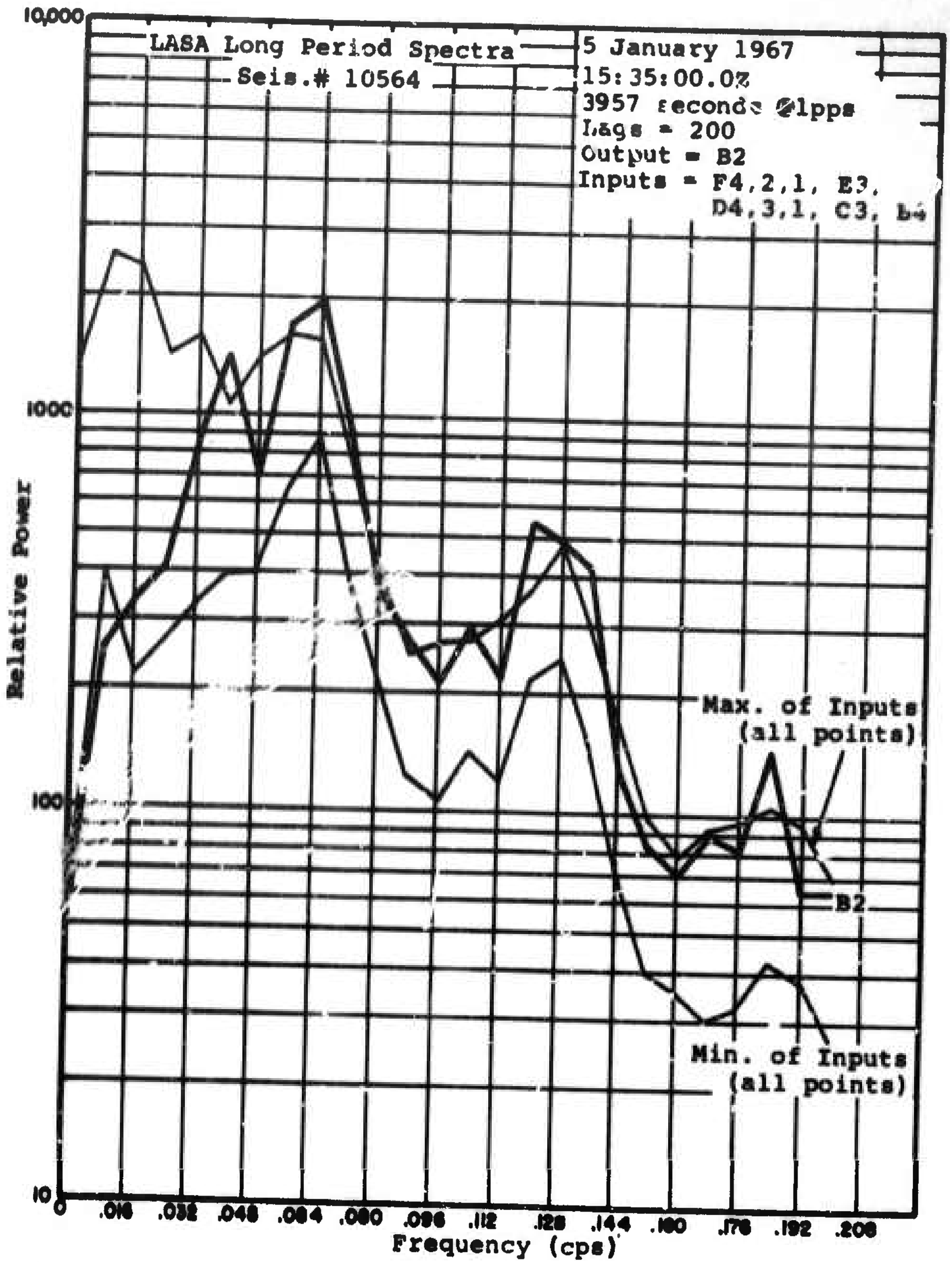


Figure 7. LASA long period spectra for a 5 Jan. 1967 sample. The figure shows the output spectrum and the range of all input spectra.

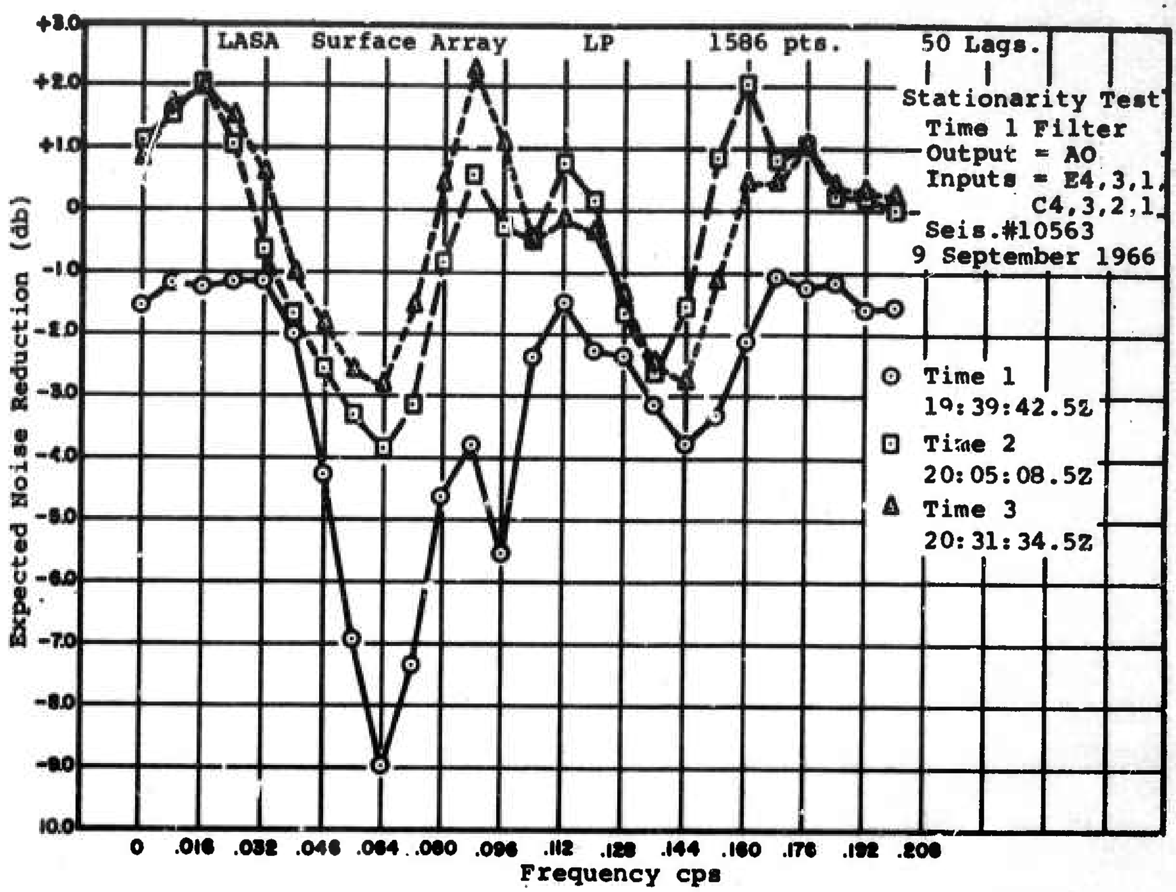


Figure 8. The expected noise reduction from a prediction error filter computer from the first time sample and applied to the first, second, and third time samples.

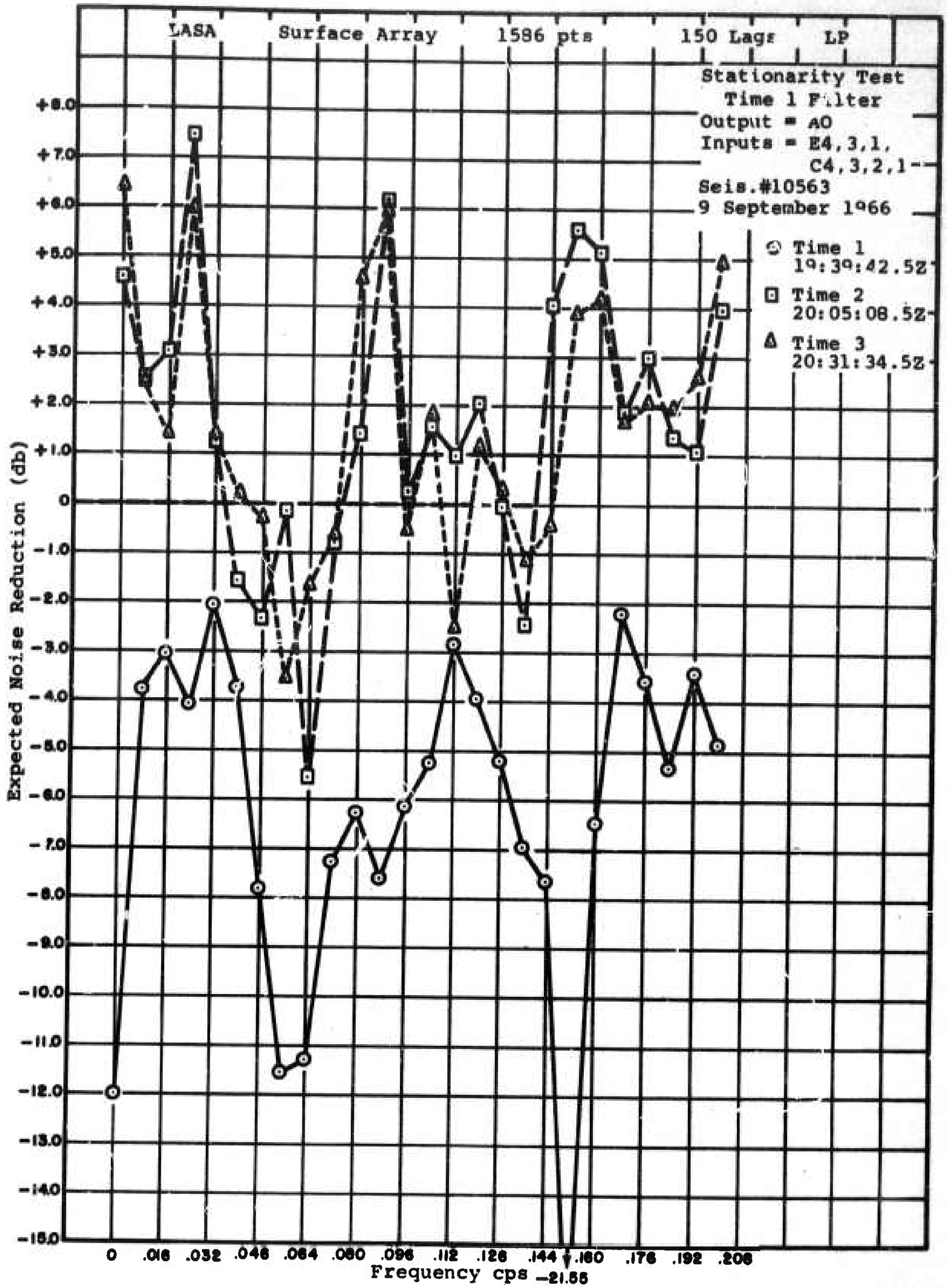


Figure 9. The expected noise reduction from a prediction error filter for the same data shown in Figure 8. Here the computations were for more lags and therefore fewer degrees of freedom.

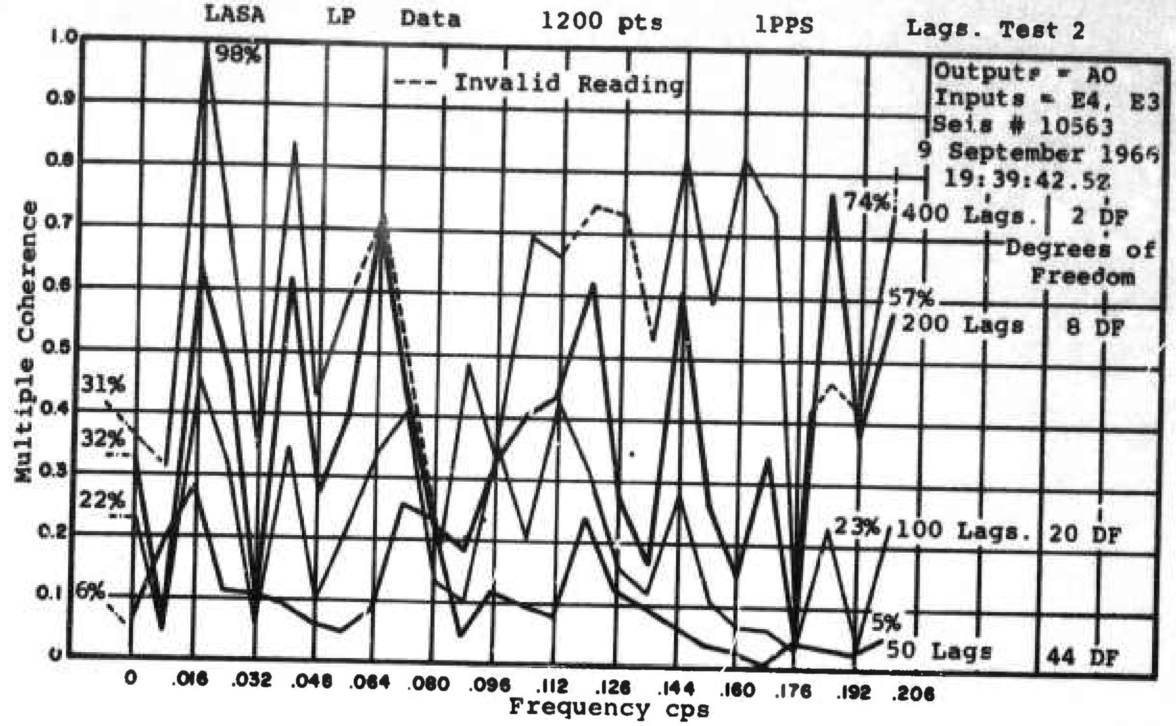


Figure 10. Multiple coherence vs. frequency with 2 inputs when the degrees of freedom are decreased from 44 to 2 in several steps by increasing the number of lags.

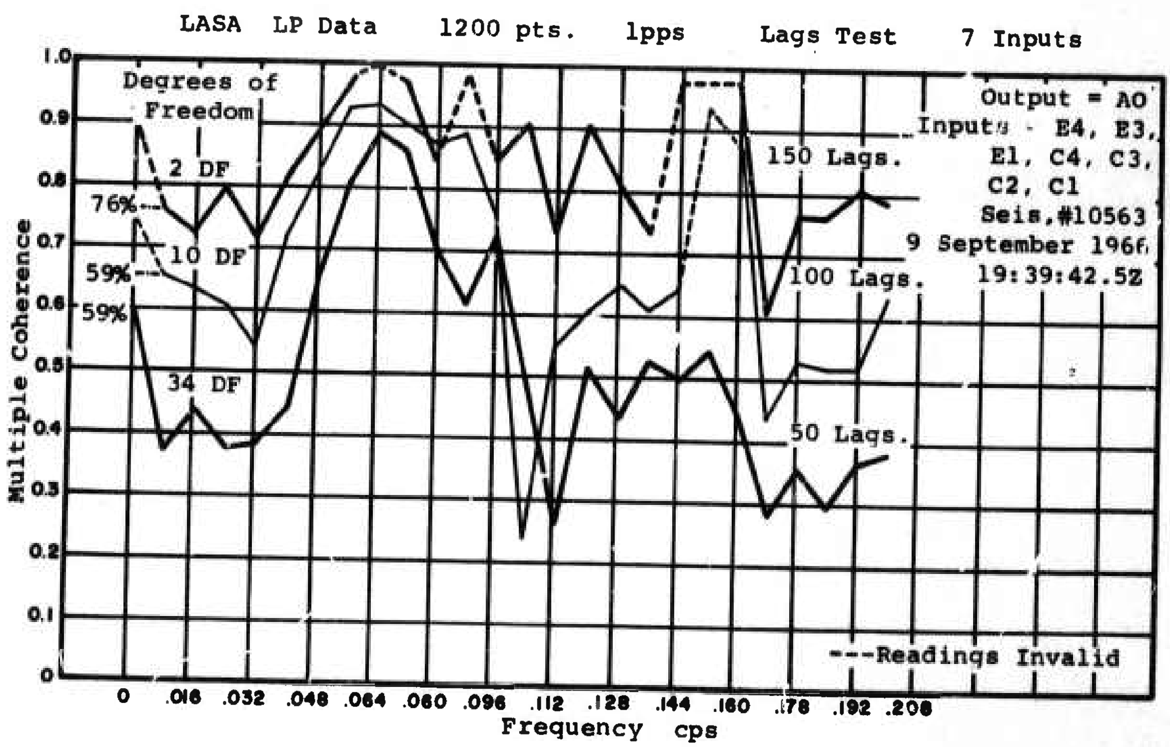


Figure 11. Multiple coherence vs. frequency with 7 inputs when the degrees of freedom are decreased from 34 to 2 in several steps by increasing the number of lags.

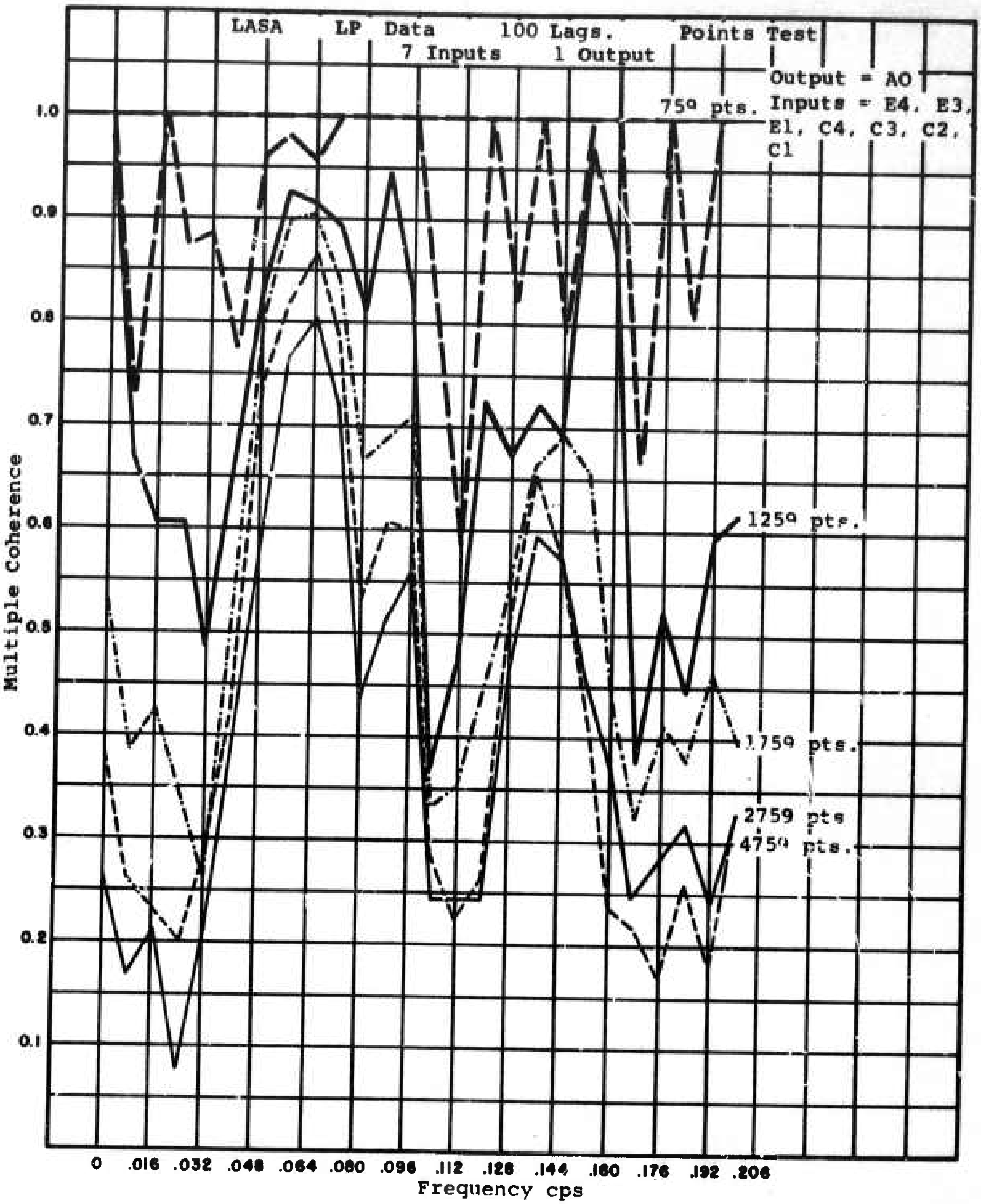


Figure 12. Multiple coherence vs. frequency with 7 inputs when the degrees of freedom are decreased by decreasing the number of points in the sample.

SEISMOGRAM # = 10563

9 September 1966

19:39:42.5Z

1200 Points

1 pps

% Coherency

Frequency	Lags 50	100	150	200	300	400
0	58.7	114.3	57.7	106.1	120.3	223.7
.008	37.6	65.7	75.7	93.6	148.9	104.1
.016	44.1	63.4	72.5	108.5	111.1	109.5
.024	38.1	61.3	79.5	96.3	100.7	59.3
.032	38.9	54.7	71.3	86.0	111.9	103.3
.040	44.3	71.8	81.4	92.1	107.9	104.0
.048	65.5	62.3	89.6	96.5	103.3	107.6
.056	80.9	92.8	96.4	99.0	97.7	101.5
.064	89.7	93.4	101.7	104.4	98.7	101.6
.072	86.2	90.6	91.4	93.4	104.1	119.4
.080	70.3	87.1	84.8	134.7	83.1	144.1
.088	61.9	88.8	129.9	90.3	65.3	116.6
.096	72.9	75.3	84.8	22.7	-89.4	282.2
.104	47.3	23.6	90.5	127.7	147.4	208.5
.112	25.7	55.0	73.1	-52.0	119.4	340.6
.120	51.7	60.8	90.6	119.2	135.4	136.1
.128	43.9	64.8	81.2	104.6	105.0	88.3
.136	52.8	61.3	73.5	45.4	94.4	62.1
.144	49.8	64.3	111.8	34.9	104.3	111.2
.152	54.9	103.8	118.7	36.4	99.6	124.5
.160	43.7	86.4	97.6	436.3	17.7	351.9
.168	28.4	43.9	60.1	111.1	46.1	41.3
.176	35.8	52.7	76.2	169.6	-2008.4	145.6
.184	29.9	51.6	76.0	142.7	90.3	92.0
.192	36.9	51.6	80.3	143.0	89.7	245.9
.200	38.2	63.3	78.5	112.1	115.9	110.2

Output = AO

Inputs = <u>E4,E3,E1,C4,C3,C2,C1</u> = 7

Frequency	Lags 50	100	150	200	300	400
0	5.7	22.3	27.5	32.5	106.7	149.0
.008	18.7	4.5	3.6	5.5	13.9	31.2
.016	28.2	44.8	55.3	63.6	71.6	98.6
.024	11.3	30.9	41.2	46.2	57.2	67.7
.032	10.9	5.1	1.9	9.8	20.2	33.3
.040	9.4	34.7	54.8	61.4	69.5	84.4
.048	6.1	9.0	17.8	26.4	42.9	43.4
.056	5.0	20.6	30.6	39.6	55.2	58.4
.064	8.4	33.6	61.1	71.3	89.3	106.8
.072	25.9	40.4	35.3	44.5	47.0	37.6
.080	23.0	12.9	18.0	22.6	24.1	10.3
.088	4.3	9.7	10.2	18.6	49.9	48.7
.096	11.5	35.4	33.9	32.9	19.8	34.0
.104	9.2	20.1	26.7	40.5	55.1	68.9
.112	8.0	41.7	43.2	43.5	45.0	67.1
.120	24.0	30.4	42.3	62.0	85.5	112.8
128	12.6	15.7	18.3	27.0	73.6	134.8
.136	9.3	11.0	13.5	16.9	32.3	53.3
.144	6.9	27.3	56.3	60.8	78.8	82.9
.152	3.7	10.5	22.4	25.7	35.7	58.4
.160	2.9	6.1	9.7	14.7	24.6	82.9
.168	0.7	6.0	17.8	34.2	68.2	74.3
.176	4.5	2.7	5.6	4.1	4.3	11.1
184	3.9	22.5	44.9	76.8	116.7	136.1
.192	2.9	3.0	19.2	37.0	38.3	41.6
.200	5.1	23.3	47.9	57.3	59.1	73.8

Output = AO

Inputs = <u>E4,E3</u> = 2

TABLE I

APPENDIX I

*Multiple Coherence Functions

Consider a collection of q clearly defined inputs $x_i(t)$; $i = 1,2,\ldots,q$, and one output $y(t)$, as pictured in Figure 5.12. Let $G_i(f) = G_{ii}(f)$ be the

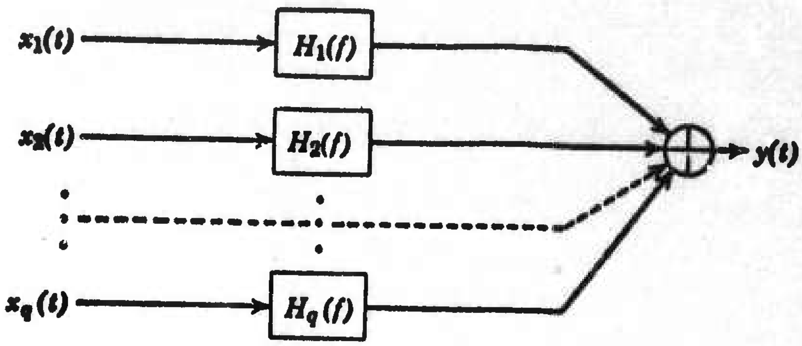


Figure 5.12 Multiple-input linear system.

power spectral density function for $x_i(t)$, and $G_{ij}(f)$ be the cross-spectral density function between $x_i(t)$ and $x_j(t)$. Define the N x N spectral matrix by

$$G_{xx}(f) = \begin{bmatrix} G_{11}(f) & G_{12}(f) & \cdots & G_{1q}(f) \\ G_{21}(f) & G_{22}(f) & & G_{2q}(f) \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ G_{q1}(f) & G_{q2}(f) & & G_{qq}(f) \end{bmatrix} \qquad (1)$$

*This explanation of multiple coherence functions was taken from "Measurement and Analysis of Random Data", Bendat, J. S., and Piersol, A. G., John Wiley and Sons, 1966. For more detailed theoretical developments and discussions of multiple, partial and marginal coherence functions, see this text.

The ordinary coherence function between $x_i(f)$ and $x_j(t)$ is defined by

$$\gamma_{ij}^2(f) = \frac{|G_{ij}(f)|^2}{G_i(f)\,G_j(f)} \tag{2}$$

The multiple coherence function between $x_i(t)$ and all other inputs $x_1(t)$, $x_2(t)$,...,excluding $x_i(t)$, is defined by

$$\gamma_{i.x}^2(f) = 1 - [G_i(f)\,G^i(f)]^{-1} \tag{3}$$

where $G^i(g)$ denotes the ith diagonal element of the inverse matrix $G_{xx}(f)^{-1}$ associated with Eq. (1). The ordinary and multiple coherence functions are both real-valued quantities which are bounded by zero and unity. That is,

$$\begin{aligned} 0 \le \gamma_{ij}^2(f) \le 1 \\ 0 \le \gamma_{i.x}^2(f) \le 1 \end{aligned} \tag{4}$$

The multiple coherence function is a measure of the linear relationship between the time history at one point, and the time histories at the collection of other points. That is, the multiple coherence function indicates whether or not the data at all of the other points linearly produce the results at a given point.

APPENDIX 2

Theoretical Development of The Stationarity Relations

A. Noise Reduction Within The Fitting Interval

A number of useful statistical measures such as ordinary and multiple coherence can be used as tools to indicate the amount of noise reduction feasible in a multiply coherent array. The basic linear model which determines the db reduction possible in the noise field by multiple coherence filtering relates a reference element (trace) $y(t)$ of an array to the other elements, say $x_1(t)$, $x_2(t)$, ..., $x_p(t)$ in the array through the linear model

$$y(t) = \sum_{k=1}^{p} \int_{-\infty}^{\infty} h_k(\alpha)\, x_k(t-\alpha)\, d\alpha \tag{1}$$

Generally we determine $h_k(t)$ as the time invariant linear filter that makes the mean square error between $y(t)$ and its predicted value a minimum, i. e.

$$E \left| y(t) - \sum_{k=1}^{p} \int_{-\infty}^{\infty} h_k(\alpha)\, x_k(t-\alpha)\, d\alpha \right|^2 = \min \tag{2}$$

which, by the usual orthogonality principle (see Papoulis 1), yields the condition

$$Ey(t)\, x_\ell(t+\tau) = \sum_{k=1}^{p} \int h_k(\alpha)\, E\, x_k(t-\alpha)\, x_\ell(t+\tau)\, d\alpha$$

$$\ell = 1,2,\ ...,\ p, -\infty<\tau<\infty \tag{3}$$

or

$$R_{yx_\ell}(\tau) = \sum_{k=1}^{p} \int h_k(\alpha)\, R_{x_k x_\ell}(\tau+\alpha)\, d\alpha \tag{4}$$

which by taking Fourier transforms implies that

$$S_{yx\ell}(\omega) = \sum_{k=1}^{p} H_k^*(\omega)\, S_{x_k x_\ell}(\omega) \tag{5}$$

Now, the mean square error can be written

$$E\left|y(t) - \sum_{k=1}^{p} \int h_k(\alpha)\, x_k(t-\alpha)d\alpha\right|^2 = E\left(y(t) - \sum_{k=1}^{p} \int_{-\infty}^{\infty} h_k(\alpha)x_k(t-\alpha)d\alpha\right) y(t)$$

$$= R_{yy}(0) - \sum_{k=1}^{p} \int h_k(\alpha)\, R_{x_k y}(\alpha)\, d\alpha$$

$$= \int_{-\infty}^{\infty} \left[S_{yy}(\omega) - \sum_{k=1}^{p} H_k^*(\omega)\, S_{x_k y}(\omega)\right] \frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} \left(S_{yy} - \underline{S}_{yx}\, S_{xx}^{-1}\, \underline{S}_{xy} \right)(\omega)\, \frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} \left(1 - \alpha^2(\omega)\right) S_{yy}(\omega)\, \frac{d\omega}{2\pi} \tag{6}$$

where $\alpha^2(\omega)$ is the multiple coherence and $(1 - \alpha^2(\omega))$ measures the reduction in power possible at the frequency ω. With $\alpha^2(\omega) = 1$ the mean square error is zero and with $\alpha^2(\omega) = 0$ the mean square error is just

$$\int_{-\infty}^{\infty} S_{yy}(\omega)\, \frac{d\omega}{2\pi} = R_{yy}(0) = E\,|y(t)|^2 \tag{7}$$

which is the original power in the process $y(t)$. Now the quantity $(1-\alpha^2(\omega))\,S_{yy}(\omega)$ represents the residual power at each frequency after the best linear estimate of the form (1) has been subtracted out. Hence the db reduction in power at each frequency is just the ratio of the output power of the residual (see equation (2)) to the input power in $y(t)$ or

$$I_H(\omega) = 10 \log \frac{S_{ee}(\omega)}{S_{yy}(\omega)} = 10 \log (1 - \alpha^2(\omega)) \tag{8}$$

where $\alpha^2(\omega)$ is the multiple coherence and $S_{ee}(\omega)$ is the power spectrum of the error process

$$e(t) = y(t) - \sum_{k=1}^{p} \int_{-\infty}^{\infty} h_k(\alpha)\, x_k(t - \alpha)\, d\alpha \tag{9}$$

B. <u>Noise Reduction Outside The Fitting Interval</u>

We would also like to determine the noise reduction in db which would result from using a set of filters $g_k(t)$, $k=1, \ldots, p$ which have been derived either from another fitting interval or from theoretical considerations. To accomplish this let $h_k(t)$ be the optimal filters for the time under investigation and let $g_k(t)$ be any other set of filters whose mean square error is to be compared with $h_k(t)$. The mean square error of the g filters can be written using the orthogonality principle as

$$E \left| y(t) - \sum_{k=1}^{p} \int_{-\infty}^{\infty} g_k(\alpha)\, x_k(t-\alpha)\, d\alpha \right|^2 =$$

$$E\left|y(t) - \sum_{k=1}^{p} \int_{-\infty}^{\infty} h_k(\alpha)x_k(t-\alpha)d\alpha + \sum_{k=1}^{p} \int_{-\infty}^{\infty} \left(h_k(\alpha) - g_k(\alpha)\right) x_k(t-\alpha)d\alpha\right|^2$$

$$= E\left|y(t) - \sum_{k=1}^{p} \int_{-\infty}^{\infty} h_k(\alpha)x_k(t-\alpha)d\alpha\right|^2 + E\left|\sum_{k=1}^{p} \int_{-\infty}^{\infty} \left(h_k(\alpha)-g_k(\alpha)\right) x_k(t-\alpha)d\alpha\right|^2$$

$$= \int_{-\infty}^{\infty} (1-\alpha^2(\omega))\, S_{yy}(\omega) \frac{d\omega}{2\pi} + \int_{-\infty}^{\infty} (\underline{H} - \underline{G})^* S_{xx}(\underline{H} - \underline{G})\ (\omega) \frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} \left[\left(1 - \alpha^2(\omega)\right) + \frac{(\underline{H} - \underline{G})^* S_{xx}\ (\underline{H} - \underline{G})\ (\omega)}{S_{yy}(\omega)} \right] S_{yy}(\omega) \frac{d\omega}{2\pi} \qquad (10)$$

Hence, if we call the new error $\ell'(t)$ we have

$$\ell'(t) = y(t) - \sum_{k=1}^{p} \int g_k(\alpha)\ x_k(t-\alpha)\ d\alpha$$

with power spectrum $S_{\ell'\ell'}(\omega)$, the new value for the improvement in the $S_k(t)$ filters would be

$$I_G(\omega) = 10 \log \frac{S_{\ell'\ell'}(\omega)}{S_{yy}(\omega)} = 10\ \log_S\left(1 - \alpha^2(\omega) + \frac{(\underline{H} - \underline{G})^* S_{xx}\ (\underline{H} - \underline{G})\ (\omega)}{S_{yy}(\omega)}\right) \qquad (12)$$

Equation (12) shows that the improvement in the $g_k(t)$ filters is expressed in terms of the improvement in the hk(t) filters and a correction term which is zero when $\underline{H} = \underline{G}$.

The improvement values $I_H(\omega)$ and $I_G(\omega)$ in equations (8) and (12) are those shown in the main body of the report.

<u>Reference</u>

1. Papoulis, A., <u>Probability, Random Variables, and Stochastic Processes</u>, McGraw Hill, 1965.

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13. ABSTRACT

Multiple coherence gives a quantitative measure versus frequency of how well a linear combination of n input channels can match the (n + 1) st channel in a seismic array. If the inputs can match the output exactly, then the multiple coherence is unity and only n channels are necessary to describe the noise field. This report shows multiple coherence versus frequency with 2 to 9 input channels for long period, vertical component noise fields at LASA.

Over the 7 to 20 seconds period range the multiple coherence on the samples tested were greater than .65 showing that 65% or more of the noise at a center channel is predictable by other seismometer outputs in the array. This level of multiple coherence requires 8 to 9 input channels. Multiple coherence with fewer inputs & ordinary coherence between pairs of channels are much lower.

From the samples tested which all produce multiple coherences quite similar to each other, we conclude that at least 9 input channels are necessary to adequately describe the long period noise at LASA.

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Multiple Coherence						
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